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Possible new definitions of the kilogram

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The kilogram is the unit of mass in the International System. Its definition dates from 1889 and, therefore, predates most of modern physics. The definition simply states that the 'kilogram' is the mass of an object known as the international prototype. Thus, we have the extraordinary situation that certain fundamental physical constants (the Planck constant, the Newtonian constant of gravitation, the mass of an electron, the mass of an atom of carbon-12, etc.) are measured in terms of an artefact that was manufactured in the nineteenth century. We begin by describing what is presently known about the long-term stability of artefact standards and how this stability can be experimentally monitored with respect to fundamental constants of physics. Finally, we suggest that present experiments are close to achieving the target uncertainty of the order 10^{-8}, although the current incoherence between two classes of experiments is cause for concern.

Keywords: kilogram; international prototype; International System of units; SI

1. Introduction

The present definition of the kilogram dates from 1889. Before discussing changes to this definition, it will be useful to examine its origin, its implications for measuring mass and related quantities in International System of units (SI) and, especially, its remarkable durability.

The metric system was first promulgated by the French republican government at the end of the eighteenth century, although the monarchy had already been working towards a similar goal (Moreau 1975). That early effort defined the kilogram as the mass of 1000 cm^3 of water at its temperature of maximum density (about 4 °C). Through more modern eyes, it is tempting to view this definition as an appeal to a physical constant (as the current definition of the kelvin appeals to another property of water, the triple-point). However, one should note that the second part of the definition relies on the metre, which was defined in terms of the Earth's circumference—not what we would consider a physical constant. In any case, the founders of the metric system knew that the definition was impractical for precise use and, therefore, needed to be embodied in a physical object. They therefore fabricated an artefact made of hot-forged platinum sponge that came to be known as the kilogram of the Archives (Moreau 1975; Davis 2003).

One contribution of 14 to a Discussion Meeting 'The fundamental constants of physics, precision measurements and the base units of the SI'.

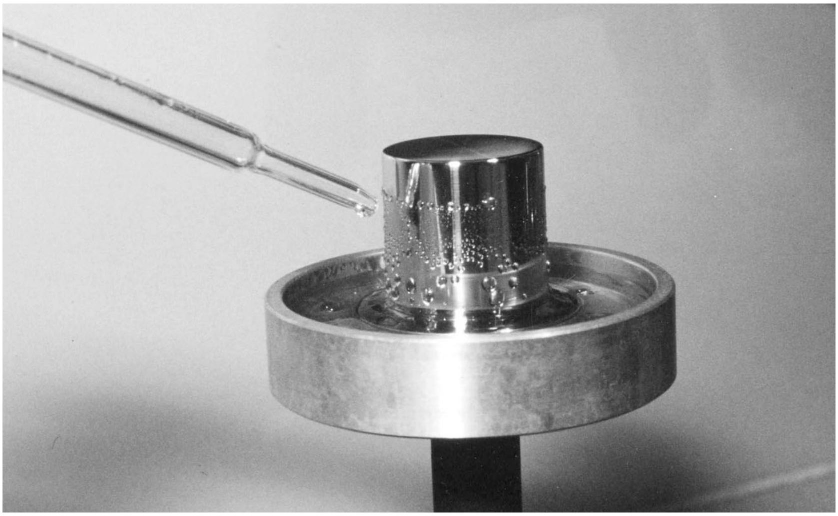

Figure 1. Copy of the international prototype being steam cleaned to remove surface contamination. Copies of the international prototype are made of the same alloy, have the same nominal dimensions and are manufactured to have the same mass (within a tolerance of ± 1 mg) as the international prototype. The mass of each copy is determined by a calibration with a relative standard uncertainty of the order 5×10^{-9}.

By the time the Bureau International des Poids et Mesures (BIPM) was created in 1875, there was no longer any thought of basing the definition of the kilogram on the density of water. Rather, the challenge was to replace the kilogram of the Archives by another artefact that took full advantage of 'modern' technology, to be known thereafter as the international prototype of the kilogram. This object is traditionally symbolized as $\mathcal{K}$. The international prototype was to have a mass that was negligibly different from that of the kilogram of the Archives but would be made of a superior alloy: 90% platinum and 10% iridium by mass (Davis 2003). Furthermore, 40 copies of this object would also be fabricated. A handful of these objects, along with $\mathcal{K}$ itself, would remain at the BIPM. The majority of the copies, to be known as national prototypes, would be attributed to Member States of the Metre Convention, the diplomatic treaty that established the entire enterprise (see http://www.bipm.org). The same system continues to this day except that additional copies of $\mathcal{K}$ have been fabricated by the BIPM until, as of this writing, the serial number has reached 91. A copy of $\mathcal{K}$ with a mass of 1 kg $\pm$ 1 mg that has been registered with the International Committee for Weights and Measures (CIPM) is referred to as a 'prototype'.

With this background, we now present the definition of the kilogram in the International System (SI; BIPM 1998):

> The kilogram is the unit of mass. It is equal to the mass of the international prototype of the kilogram.

This is essentially the definition of 1889 and we should remember that a similar artefact-based reference defined the metre until 1960. That the international prototype of the kilogram and its copies are indeed artefacts can be seen in figure 1.

According to the definition, the mass m_X in kilograms of any object X is given by

$$m_X = \left\{\frac{m_X}{m_{\mathcal{K}}}\right\}[\mathrm{kg}], \tag{1.1}$$

where we use the notation of quantity calculus (de Boer 1994/95) to show the SI unit in square brackets and the number associated with the unit in curly brackets. The comparison of an object with respect to $\mathcal{K}$ is carried out by weighing on precise mass comparators (Quinn 1992; Gläser 2000; Picard 2004). Of course, it is not practical to weigh all objects of interest directly against $\mathcal{K}$. Instead, a hierarchy of mass standards has been established. From time to time, the national prototypes are brought to the BIPM for comparison with $\mathcal{K}$. It is then the job of each member state to disseminate this calibration to secondary mass standards, usually made of nonmagnetic stainless steel, with nominal values from 1 mg or below to many tons (Bich 2003). Schematically, the real calibration chain can be represented as (Davis 2001)

$$m_X = \left\{\frac{m_X}{m_n}\right\}\left\{\frac{m_n}{m_{n-1}}\right\}\cdots\left\{\frac{m_2}{m_1}\right\}\left\{\frac{m_1}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]. \tag{1.2}$$

Every ratio in curly brackets represents a measurement with an associated uncertainty and, obviously, correlations among the series of measurements must be taken into account. Ultimately, an end user working in science, commerce or other domains requires the mass of quantities that are *not* mass standards. Mass standards are useful only to the extent that they provide the link from such quantities to $m_{\mathcal{K}}$.

The utility of equation (1.1) is premised on the belief that the mass of Pt–Ir alloy is chemically inert and that, therefore, the mass of a 1 kg prototype is stable in time as long as it is handled with care. Of course the surface of any object will become contaminated after a prolonged exposure to air. Therefore, it is now understood that the definition of the kilogram given above applies immediately after cleaning by a prescribed technique (Girard 1990, 1994) that has been demonstrated not to remove Pt–Ir alloy but which is thought to remove physisorbed contamination (Davidson 2003). Suppose, for instance, that $\mathcal{K}$ is cleaned at time $t = t'$. It has been observed (Girard 1994) that

$$\left\{\frac{m_X}{m_{\mathcal{K}}}\right\}_{t=t'-\varepsilon}\left\{\frac{m_{\mathcal{K}}}{m_X}\right\}_{t=t'+\varepsilon} < 1, \tag{1.3}$$

where ε represents a time period of several days to several weeks and m_X is the average mass of an ensemble of prototypes that have not been recently cleaned. This average mass is thought to be stable over a period of at least 2ε. To cope with the inequality shown in equation (1.3), the CIPM has mandated that the mass of $\mathcal{K}$ be taken as exactly 1 kg at a time just after the prescribed method of cleaning. In practice, several cleaning operations were carried out in succession before the mass of $\mathcal{K}$ was found to have stabilized (Girard 1994). We have assumed that all surface contamination is removed by cleaning. However, there is at least one exception. Cumpson & Seah (1995) have shown that mercury vapour adheres to a platinum surface and is not removed by normal cleaning methods.

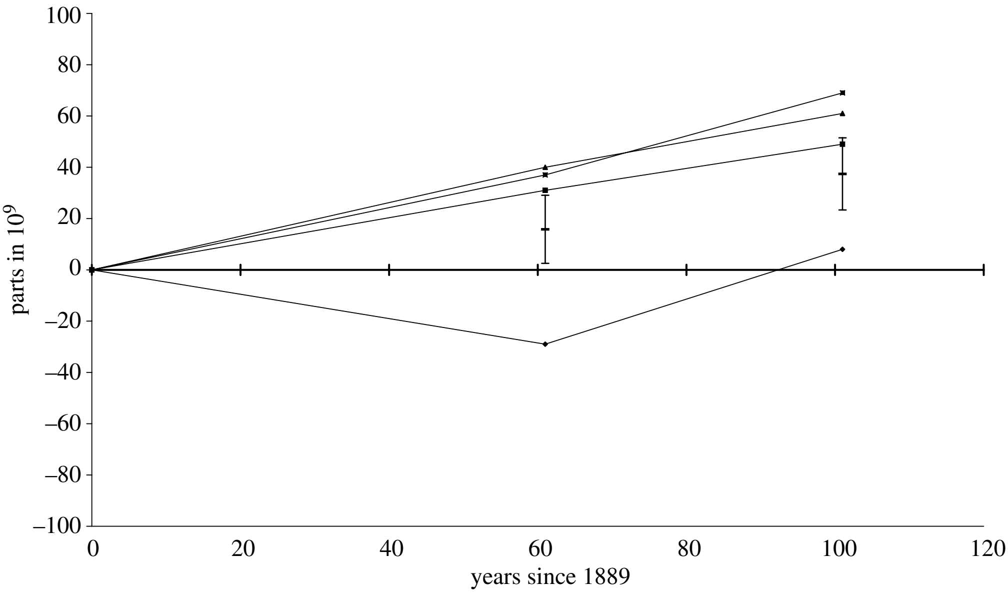

Figure 2. Mass with respect to the international prototype of four official copies. The vertical axis represents $\left(\left\{\frac{m_{X_i}}{m_{\mathcal{K}}}\right\}_t - \left\{\frac{m_{X_i}}{m_{\mathcal{K}}}\right\}_0\right)\Big/10^{-9}$, where X_i represents the i^{th} of the five artefacts.

2. Stability of the prototypes

From time to time, one may read accounts in the popular press reporting that the international prototype is 'losing mass' or 'getting lighter'. Such statements beg the question (never addressed): compared with what? The inference is: compared with a more stable mass. If such assertions could be made with certainty, it would be a rather simple matter to replace the international prototype with this 'more stable' mass.

And yet the assertion has some basis in fact. The international prototype is stored with its six official copies. Four of these copies were put into service in 1889 along with the international prototype itself. Now it might be argued that the average mass of these five objects should be more stable (compared with the mass of ^{12}C atom, for instance) than the mass of any single artefact. Relative changes in the mass of the official copies compared with the international prototype are shown in figure 2. The time axis is years since 1889. In the notation adopted in equation (1.1), the points on the vertical axis represent

$$\left(\left\{\frac{m_{X_i}}{m_{\mathcal{K}}}\right\}_t - \left\{\frac{m_{X_i}}{m_{\mathcal{K}}}\right\}_0\right)\Big/10^{-9}, \qquad (2.1a)$$

where X_i is any of the five artefacts, including the international prototype, and t is the number of years since 1889. As can be seen from the graph, measurements were carried out over three periods, one at about $t=0$, the second at about $t=61$ years and the third at about $t=101$ years. The standard uncertainty of each term in equation (1.2) is roughly 3×10^{-9} for the first and third series and perhaps more for the second (Davis 2003). These uncertainties are not shown in the

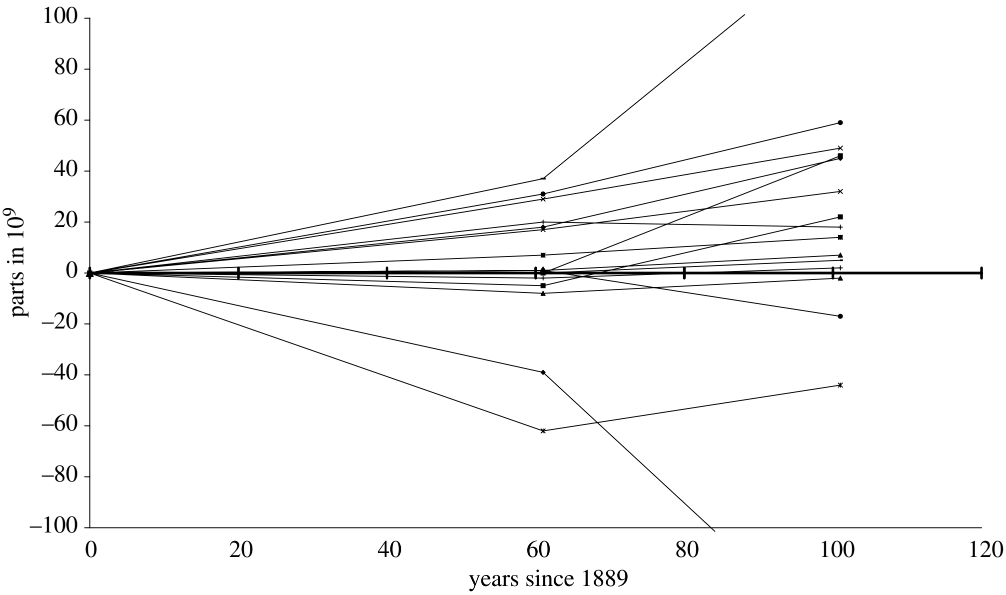

Figure 3. Mass with respect to the international prototype of 16 national prototypes. The vertical axis represents $\left(\left\{\frac{m_{X_i}}{m_{\mathcal{K}}}\right\}_t - \left\{\frac{m_{X_i}}{m_{\mathcal{K}}}\right\}_0\right)\Big/10^{-9}$, where X_i represents the i^{th} of the 17 artefacts.

figure. Lines have been drawn to connect the data in order to make trends more apparent. For the second and third series, we show the mean of the five data points along with its standard uncertainty. Based on these considerations, one could argue that the mass of the international prototype is decreasing with respect to the mean of an ensemble that includes the four oldest official copies and that a reasonable inference is that the mass of the international prototype is also decreasing with respect to the mass of an atom of ^{12}C. This argument is far from being a proof.

The inference becomes stronger if one then looks at a similar graph drawn for the national prototypes that have been calibrated three times with respect to the international prototype (figure 3). The dispersion is much greater as would be expected since these artefacts have been stored and used under very different conditions and most have been transported over large distances. Although mechanisms leading to a mass decrease are obvious (wear, accident), we see that the general trend is an increase in mass with time, with respect to the international prototype. The median of these results has increased to +16 μg by the third set of data. The averages and medians from figures 2 and 3 are summarized in table 1, where we have used the suggestion of Müller (2000) to estimate the standard deviation of the median.

While these ensemble averages and medians might be used to replace the present definition of the kilogram, the dispersion of the individual points is a concern. In a few extreme cases, outliers were identified *a priori* by unusual changes to the surface of the artefact but, in most cases, there is no satisfactory explanation for the dispersion. The situation is reminiscent of the days when national standards of voltage were maintained by electrochemical devices known as Westin cells. Various techniques were employed to create the most stable

Table 1. *Averages and medians from data shown in figures 1 and 2*

	t=61 years	t=101 years
figure 2		
mean	15.8	37.4
s.d. of mean	13.3	14.1
median	31	49
s.d. of median	7.6	17.0
figure 3		
mean	2.6	13
s.d. of mean	5.9	13.6
median	1	16
s.d. of median	6.1	11.5

ensemble of cells but these efforts were found to have been problematic once a truly stable quantum voltage standard, based on the Josephson effect, was developed (Cohen & Taylor 1973; Melchert 1978). Ensemble averages do have their uses, as in the case of International Atomic Time (TAI; Petit 2003).

A second reason why ensemble averages have not been used to redefine the unit of mass is that the present system has served the needs of metrology rather well. As a practical matter, stainless steel weights are bought and sold based on tolerance classes established by the International Organization of Legal Metrology (OIML). The highest quality is class E_1, for which the manufacturing tolerance for 1 kg artefacts is 0.5 mg (5 parts in 10^7; OIML 1994; Källgren & Pendrill 2003). Such weights are found in the catalogue of major manufacturers. To support this system, the national metrology institutes (NMIs) of many countries offer routine calibration services. Typically, calibrations of 1 kg stainless steel standards can be carried out with an expanded uncertainty (95% confidence level) of about 0.05 mg (5 parts in 10^8).

As shown in figures 2 and 3, the dispersion among prototypes increases with time. It is, therefore, useful to ask whether determinations of physical constants made over a time-scale of many decades are consistent within their combined uncertainties. For instance, one might look at determinations of the density of distilled water made at the beginning and end of the twentieth century, or the density of a sample of mercury that was standardized in the 1950 s and then used in the 1980 s, to establish a link to the SI volt (Davis 1989, 2001).

It is important to be clear about what one means by the statement that the mass of the international prototype can be changing with time. We have already presented examples in equations (1.3) and (2.1a). We may rewrite equation (2.1a) so that it refers to two measurements of some mass m_X separated by a time-interval (t_2-t_1), typically dozens of years

$$\left(\left\{\frac{m_X}{m_{\mathcal{K}}}\right\}_{t=t_1}-\left\{\frac{m_X}{m_{\mathcal{K}}}\right\}_{t=t_2}\right)\Big/(t_2-t_1)=\alpha_m. \tag{2.1b}$$

The SI would attribute a finite value of the coefficient α_m to a change in m_X over the time period (t_2-t_1). However, this interpretation would be highly questionable if, for example, $m_X=m(^{12}C)$. If m_X were the mass of a known volume of distilled water or mercury, a finite value of α_m might also lead us to question the suitability of $m_{\mathcal{K}}$ as the best available reference for the unit of mass. An examination of available data does not reveal a finite value for α_m but the relative standard uncertainty of the estimate is not very useful: about 19×10^{-9} per year. That such tests have proved to be negative simply indicates that the precision of $m_X/m_{\mathcal{K}}$ determinations have not achieved sufficient precision for the intervals (t_2-t_1) that have been probed.

3. Further implications of the artefact definition of the kilogram

(*a*) *Atomic masses*

The artefact definition of the kilogram greatly affects two additional base units of the SI: the mole and the ampere.

As soon as the atomic view of matter became well established, it was recognized that the artefact kilogram is not an appropriate standard for establishing a scale of atomic and subatomic masses. We illustrate this with the examples of the mass of an atom of carbon-12, $m(^{12}\mathrm{C})$ and the mass of an electron, m_e. In the SI

$$m(^{12}\mathrm{C}) = \left\{\frac{m(^{12}\mathrm{C})}{m_{\mathcal{K}}}\right\}[\mathrm{kg}], \tag{3.1}$$

$$m_e = \left\{\frac{m_e}{m(^{12}\mathrm{C})}\right\}\left\{\frac{m(^{12}\mathrm{C})}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]. \tag{3.2}$$

The ratio on the right-hand side of equation (3.1) is approximately 1.66×10^{-27}. This mismatch of 27 orders of magnitude represents an experimental challenge. The present relative standard uncertainty of this measurement, which will be discussed in §6, is 1.7×10^{-7}. Equation (3.2) has been written so that the correlation with equation (3.1) through the mass of the international prototype is obvious. In fact, the mass of the electron with respect to the mass of an atom of carbon-12 is now known to a relative standard uncertainty of 4.4×10^{-10} (Mohr & Taylor 2005).

Owing to these considerations, atomic masses are routinely reported in terms of the unified atomic mass unit, u, defined as (Petley 1989)

$$1[\mathrm{u}] = \frac{1}{12}m(^{12}\mathrm{C}) = \frac{1}{12}\left\{\frac{m(^{12}\mathrm{C})}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]. \tag{3.3}$$

When expressed in terms of the non-SI unit of mass u (instead of the SI unit of mass kg), $\mathrm{m}(^{12}\mathrm{C})$ is an exactly defined quantity and, consequently, the value of m_e expressed in u has a relative uncertainty of 4.4×10^{-10}. Alternatively, and staying within the SI, one can compute the uncertainty of m_e, measured in terms of $m(^{12}\mathrm{C})$, by taking account of the correlations between the measurements of the relevant fundamental constants (Mohr & Taylor 2005). Making use of the covariance matrix that links the various fundamental constants is an alternative way of carrying out the calculation.

In 1972, the mole was added as a base unit of the SI: "The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12;" (BIPM 1998). The Avogadro constant, N_A, is intimately related to this definition through the relation

$$1\ \mathrm{u} = \frac{1}{N_\mathrm{A}} 10^{-3}\ \mathrm{kg\ mol^{-1}}. \tag{3.4}$$

Thus, we might seek a new definition of the kilogram based on fixing the value of $m(^{12}\mathrm{C})$ in equation (3.3) or, equivalently, fixing the value of the N_A in equation (3.4). In either case, the challenge would be to measure the ratio in the right-hand side of equation (3.1) with an uncertainty that sheds light on the experimentally observed dispersion of artefact mass standards; as shown, for example, in figures 2 and 3. An influential paper by Quinn (1991) proposed a relative uncertainty of the order 10^{-8} as the target and this has been widely accepted.

(*b*) *Electrical units*

One need only glance at the SI definition of the ampere to see that the kilogram is implicated in electrical units: "The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length" (BIPM 1998). Of course the Newton, with units of kg m $\mathrm{s^{-2}}$, is measured in terms of the international prototype.

The ampere definition implies the following equation

$$F_\mathrm{L} = \frac{\mu_0}{2\pi}\frac{i^2}{d}, \tag{3.5}$$

where i is 1 A, d is 1 m and μ_0 is the magnetic constant, the value of which is fixed at $4\pi\times10^{-7}\ \mathrm{N\ A^{-2}}$. The left-hand side of equation (3.5) is the force per unit length of conductor.

This definition dates from 1948 and, therefore, precedes the development of quantum electrical standards by many years (Piquemal *et al.* 2004); principally voltages derived from the Josephson effect (Pöpel 1992) and resistances derived from the quantum Hall effect (Jeckelmann & Jeanneret 2001). In fact, if the current in equation (3.5) were measured in terms of these quantum standards, equation (3.5) would be transformed to

$$F_\mathrm{L} = \frac{\mu_0 e^2}{8\pi d}(i_{90})^2(K_{J-90}R_{K-90})^2, \tag{3.6}$$

where e is the elementary charge, i_{90} is the conventional current (non-SI) routinely measured by laboratories using conventional quantum electrical standards and K_{J-90} and R_{K-90} are conventionally defined constants, discussed further in §5. Thus, the elementary charge is also related to the artefact definition of the kilogram. Indeed, the kilogram could, in principle, be redefined by fixing the value of the elementary charge.

Other electrical quantities can also be measured in terms of the international prototype. Among these, the most interesting is electrical power, which will be discussed in more detail below. Here we merely point out that electrical power $(\mathrm{V^2\,\Omega^{-1}})$, P, when measured in terms of conventional quantum electrical standards, becomes

$$P = \frac{h}{4} P_{90}(K_{J-90}^2 R_{K-90}), \tag{3.7}$$

where h is the Planck constant and P_{90} is the non-SI electrical power measured in terms of conventional quantum electrical standards. The quantity P is the power in SI units and, therefore, depends on the kilogram. Thus, fixing h is also a means of redefining the kilogram. At present, however, an experiment to determine electrical power, P_{90}, measured with conventional quantum standards while simultaneously measuring the same power in SI units is a determination of h (in SI units). This type of experiment, known as a 'watt balance', is discussed in §5.

Finally, we point out that equations (3.6) and (3.7) contain elements of the fine-structure constant

$$\alpha = \frac{\mu_0 e^2 c}{2h}, \tag{3.8}$$

where c is the speed of light in vacuum which is, of course, a fixed quantity (BIPM 1998; Mohr & Taylor 2005). Since one is not free to choose an arbitrary value for α, it is impossible to create a unit system that fixes the values of all four physical constants on the right-hand side of equation (3.8).

4. International Avogadro coordination

The link between atomic mass and the artefact kilogram is being pursued by a working group established at the request of the CIPM. This effort is being coordinated by the Physikalisch-Technische Bundesanstalt (the NMI of Germany) and includes research groups from the NMIs of Australia, Italy, Japan, the UK and the USA as well as the Institute for Reference Materials and Measurements, the BIPM and others. The diversity of effort reflects the need for many disciplines, not all of which are available in one institute. The basic idea and current status of the working group have been described in recent review papers (Becker 2003; Schwitz *et al.* 2004) and need not be repeated. Here we simply put the effort in the context of the formalism developed above.

The experiment is based on the properties of single-crystal silicon. Silicon has three naturally occurring isotopes but, conceptually, it is easier to consider a perfect single crystal of chemically and isotopically pure ^{28}Si. One starts by fabricating a highly perfect 1 kg sphere of this material. The challenge is then to determine the number n of ^{28}Si atoms in the sphere. If the crystal is perfect, the isotopic and chemical impurities negligible and the surface uncontaminated, then

$$n = \left\{ \frac{m_{\text{sphere}}}{m(^{28}\text{Si})} \right\}, \tag{4.1}$$

where m_{sphere} is the mass of the sphere. It is useful to rewrite equation (4.1) in such a way that the mass of the international prototype and the mass of an atom of ^{12}C are explicit

$$n = \left\{ \frac{m_{\text{sphere}}}{m_{\mathcal{K}}} \right\} \left\{ \frac{m_{\mathcal{K}}}{m(^{12}\text{C})} \right\} \left\{ \frac{m(^{12}\text{C})}{m(^{28}\text{Si})} \right\}. \tag{4.2}$$

The first ratio on the right-hand side of equation (4.2) represents a comparative weighing of two artefacts with a nominal mass of 1 kg. As such, it does not represent an insurmountable challenge at a relative uncertainty of 10^{-8}. The international prototype appears in equation (4.2) for heuristic reasons. One may use any mass that is linked to $m_{\mathcal{K}}$ with sufficiently small uncertainty. The third ratio is that of two atomic masses and it can also be carried out to high precision using ion trapping or similar techniques. As in equation (3.1), the middle ratio represents a mismatch of 27 orders of magnitude and, therefore, cannot be directly determined with high accuracy.

However, there is a second route to determining n. It is the ratio of the volume (V) of the sphere to the volume of a ^{28}Si atom within the sphere

$$n = 8\frac{V}{a^3}, \tag{4.3}$$

where a unit cell of side a contains eight atoms. A measurement of the ratio V/a^3 can attain a sufficiently low uncertainty (Becker 2003). The diameter of the sphere can be related to the wavelength of a stabilized laser by means of an optical interferometer and a can, in principle, be related to the same optical wavelength by means of a single-crystal X-ray interferometer. Producing the silicon in the form of a nearly perfect sphere permits an accurate determination of V from optical interferometry. Unless one is willing to destroy the sphere, it is not possible to make all the necessary measurements on the same piece of silicon. Thus, homogeneity of the silicon used in the experiment is a major consideration.

Rearranging terms in equations (4.2) and (4.3) gives

$$\left\{\frac{m(^{12}\mathrm{C})}{m_{\mathcal{K}}}\right\} = \left\{\frac{m_{\mathrm{sphere}}}{m_{\mathcal{K}}}\right\}\left\{\frac{m(^{12}\mathrm{C})}{m(^{28}\mathrm{Si})}\right\}\frac{a^3}{8\,V}. \tag{4.4}$$

Based on equations (3.3) and (3.4), we can multiply both sides of equation (4.4) by a defined constant so that the experiment now looks like a measurement of N_{A}

$$\frac{1}{N_{\mathrm{A}}} = \left\{\frac{m(^{12}\mathrm{C})}{m_{\mathcal{K}}}\right\}\frac{10^3}{12}[\mathrm{mol}]. \tag{4.5}$$

We emphasize that the above derivation assumed that the crystal is made of pure ^{28}Si. To date, progress in the experiment has been limited in large part by the uncertainty in the measured isotopic-abundance ratios of the three naturally occurring silicon isotopes used to grow the single-crystal samples. The Avogadro working group is well along in its project to acquire samples of highly enriched ^{28}Si, which will significantly reduce the present uncertainty.

Using natural material, the working group has arrived at the following value for N_{A} (Mohr & Taylor 2005)

$$\begin{aligned} N_{\mathrm{A}} &= 6.022\,1353\times 10^{23}\ \mathrm{mol}^{-1} \\ u_{\mathrm{r}}(N_{\mathrm{A}}) &= 3.2\times 10^{-7} \end{aligned},$$

where u_{r} indicates the relative standard uncertainty of the result.

With crystals made of enriched ^{28}Si, the working group projects a relative standard uncertainty of about 2×10^{-8} in N_A (Becker 2003).

5. Watt balances

The basic principle of the watt balance was set forth by Kibble of the National Physical Laboratory (NPL) in the final section of a 1975 conference paper, which was published the following year (Kibble 1976). However, in 1975, the proposed experiment was seen as a way to determine the SI ampere in terms of the kilogram. Since the perfection of quantum electrical standards, the interpretation of the experiment has evolved to become a determination of the Planck constant h in terms of the kilogram.

The NPL soon undertook the proposed experiment, followed by the National Institute of Standards and Technology (NIST). More recently, a watt balance has been constructed by the Swiss Office for Metrology and Accreditation. The Laboratoire National de Métrologie is well along in constructing their own watt balance and the BIPM apparatus in the design stage. The interested reader may consult excellent recent review papers discussing the design, operation and performance of these instruments (Eichenberger *et al.* 2003; Schwitz *et al.* 2004). As with the Avogadro experiment, it will be useful to follow a schematic development of a watt balance experiment. Here we have selected that of the NIST because this experiment has produced the most recent published value of h (Mohr & Taylor 2005).

The basic experiment is shown in figure 4. The movable induction coil is suspended from a balance. At the position of the coil, there is a radial magnetic induction $B(r)$ produced by two opposing superconducting solenoids. Note that the balance 'beam' is actually a wheel in the NIST design. Near the top of the coil suspension is a 1 kg mass standard that can be placed on or off the suspension.

In the first part of the experiment, the 1 kg standard is removed and the auxiliary drive coil is used to rotate the balance wheel so that the induction coil is displaced within the uniform magnetic field at constant velocity, v. The advantage of the wheel is that the effective arm-length of the balance is independent of the rotation angle. The induction coil is open-circuited so that a voltage U is induced at the ends of the coil

$$U = vB(r)\pi D n_c, \tag{5.1}$$

where D is the diameter of the coil, n_c is the number of turns and v is the velocity of the induction coil. Note that equation (5.1) is highly simplified in that it assumes that the geometry of the coil and field are perfect and that the voltage and velocity can be measured instantaneously. It would be a daunting task to measure the geometric terms and the magnetic induction to a high level of accuracy.

In the second part of the experiment, the mass of the 1 kg standard is balanced by the Lorentz force produced by running a current I through the induction coil

$$m_{\mathcal{K}} g = IB(r)\pi D n_c, \tag{5.2}$$

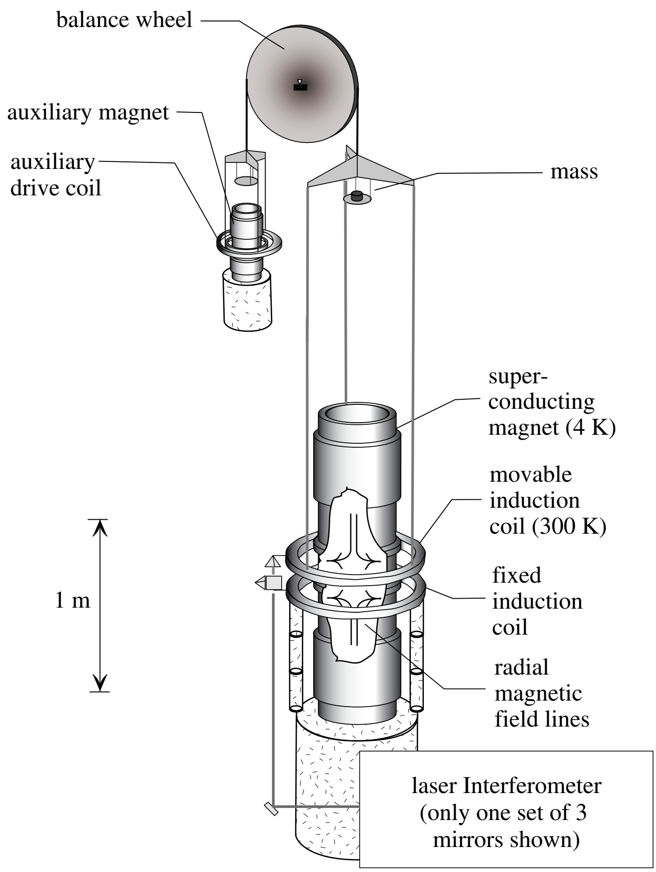

Figure 4. Schematic of the NIST watt balance. Reprinted with permission from Williams *et al*. 1998 (Copyright © American Physical Society).

where g is the local acceleration of gravity and we have, for illustrative purposes, assumed that the international prototype has been placed on the balance. Of course a different mass standard, directly traceable to $\mathcal{K}$, would normally be used.

As first recognized by Kibble, doing the experiment in this way eliminates the problematic magnetic induction and geometric terms. This is much easier said than done as the reader may appreciate from reading detailed analyses of the experiment (e.g. Gillespie *et al*. 1997). One then arrives at the result

$$m_{\mathcal{K}} g v = (IU)_{90}, \tag{5.3}$$

hence the name 'watt balance'. The left-hand side of equation (5.3) contains quantities that are measurable to sufficient accuracy in terms of SI base units of mass, length and time. However, as we now show in equation (5.3), the electrical terms on the right-hand side are measured in terms of the 1990 conventions for quantum electrical references of voltage and resistance. These laboratory measurements are only approximate SI values of voltage and resistance since they are defined through *conventions* adopted in 1990 rather than from the SI definition of the ampere (Taylor & Witt 1989). This complication can be dealt with in the following way (Mohr & Taylor 2005). The conventional representation of voltage, V_{90}, is related to the SI unit V by

$$V_{90} = \left\{ \frac{K_{J-90}}{K_J} \right\} [\mathrm{V}], \tag{5.4}$$

where K_{J-90} is the conventionally chosen value of the Josephson constant and K_J is its SI value, $2e/h$. Note that K_{J-90} has no uncertainty. Similarly, the conventional unit of resistance, Ω_{90}, is related to the SI ohm Ω through the relation

$$\Omega_{90} = \left\{ \frac{R_K}{R_{K-90}} \right\} [\Omega], \tag{5.5}$$

where R_{K-90} is the conventional value of the von Klitzing constant and R_K is its SI value, h/e^2. Note that R_{K-90} has no uncertainty. Thus, equation (5.3) becomes

$$m_{\mathcal{K}} = h \frac{1}{4gv} \left(\frac{U' U}{R} \right)_{90} K_{J-90}^2 R_{K-90}, \tag{5.6}$$

where $U'/R = I$ and the quantity in parentheses is measured in terms of 1990 conventional quantum electrical references.

At the time of writing, there are two available values of h derived from watt balance experiments, a 1990 value from NPL and a 1998 value from NIST. They are discussed in the 2002 CODATA recommended values of the fundamental constants as reported by Mohr & Taylor (2005):

NPL.
$h = 6.626\,0682 \times 10^{-34}$ Js; $u_r(h) = 2.0 \times 10^{-7}$;

NIST.
$h = 6.626\,068\,91 \times 10^{-34}$ Js; $u_r(h) = 8.7 \times 10^{-8}$.

The agreement between the two results is well within their combined uncertainty.

6. Redundancy

A useful redundancy exists among the physical constants. For instance,

$$\frac{h}{m_e} = \frac{c\alpha^2}{2R_\infty}, \tag{6.1}$$

where R_∞ is the Rydberg constant (Mohr & Taylor 2005). The relative uncertainty of the right-hand side of equation (6.1) is dominated by the standard relative uncertainty of the square of the fine-structure constant $u_r(\alpha^2) = 6.7 \times 10^{-9}$. Thus, at a relative uncertainty of the order 1×10^{-8}, the watt balance provides a measure of $m_e/m_{\mathcal{K}}$. More interestingly, the Avogadro constant can be written as

$$N_A = 0.012 \frac{1}{m(^{12}\mathrm{C})} = 0.012 \frac{1}{m(^{12}\mathrm{C})} \frac{m_e}{m_e} = 0.012 \left\{ \frac{m_e}{m(^{12}\mathrm{C})} \right\} \frac{1}{m_e}. \tag{6.2}$$

Expressed in this way, the ratio in brackets is simply the electron mass relative to the mass of an atom of carbon-12. We have already seen that this ratio has a relative standard uncertainty of 4.4×10^{-10}. Combining equations (6.1) and (6.2) to eliminate the common factor of m_e gives

$$N_A h = 0.012\left\{\frac{m_e}{m(^{12}C)}\right\}\frac{c\alpha^2}{2R_\infty}. \tag{6.3}$$

Evidently, the relative standard uncertainty of the right-hand side of equation (6.3) is 6.7×10^{-9}. To this uncertainty, a measurement of N_A can be compared with a measurement of h in order to check the consistency of these two independent links to the international prototype. If we take the value of N_A as put forward by the Avogadro working group and convert it to h via equation (6.3), the result is (Mohr & Taylor 2005):

Avogadro (2002 CODATA):
$h = 6.626\,0762\times10^{-8}$ J s; $u_r(h) = 3.2\times10^{-7}$

In relative terms, the present discrepancy between, for example, h determined from the NIST watt balance and h determined via equation (6.3) from the value of N_A reported by the Avogadro working group is 1.1×10^{-6}. The discrepancy far exceeds the combined *a priori* relative standard uncertainty of each experiment, 0.33×10^{-6}. Faced with this discrepancy, 2002 CODATA makes the following recommendation

Recommended (2002 CODATA):
$h = 6.626\,0693\times10^{-8}$ Js; $u_r(h) = 1.7\times10^{-7}$

We point out that a discrepancy of 10^{-6} is large compared with the observed dispersion of artefact 1 kg prototypes over the course of 100 years and, indeed, is large compared with the standard uncertainties routinely available at the 1 kg level from NMIs.

A recent article by Mills *et al.* (2005) proposes that either the value of N_A or that of h be defined in 2007 as that given in the 2002 CODATA recommended values of the fundamental constants. If the proposal is accepted, the mass of the international prototype would become an experimentally determined quantity. The mass of the international prototype would thereafter be determined through watt balances, Avogadro determinations or similar experiments. Wignall (2005) has recently made a similar suggestion for redefining the kilogram. These proposals, while attractive to physicists dealing with fundamental constants, are largely irrelevant to the problem discussed above—that of establishing and maintaining the link between a 1 kg artefact and a fundamental constant to a relative uncertainty of about 10^{-8}. The proposal would simply reinterpret the experiments aimed at establishing this link so that the existing experimental uncertainty would be associated with the SI mass of the international prototype.

7. Conclusion

There is clear experimental evidence that the masses of 1 kg prototypes are not stable when compared among themselves over many dozens of years. To date, however, there is no experimental evidence to support a hypothesis that the mass of the international prototype is changing with respect to a mass derived from a fundamental constant of physics. The current lack of such evidence is presumably due to the present level of precision of watt balance and Avogadro experiments.

The current discrepancy between the Planck constant derived from watt balances and the Planck constant derived from the international Avogadro project is about 10^{-6}. This is significant when compared with the stated uncertainties of these results. The Avogadro group is well along in a programme aimed at significantly reducing their current uncertainties. Five laboratories are working on watt balances and two of these laboratories have already published results for Planck's constant.

An uncertainty of a few parts in 10^8 in watt balance or Avogadro experiments would allow us to monitor the stability of the international prototype at a useful level, thus hastening the day when all users of the SI will welcome a redefinition of the kilogram based on a fundamental constant of physics. We emphasize that, after such a redefinition, the inherent instability of artefacts implies that at least one experiment (and, ideally, several) linking the international prototype to a fundamental constant should be maintained.

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