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FUNDAMENTAL PROBLEMS IN METROLOGY

POSSIBLE DEFINITION OF THE UNIT OF MASS AND FIXED VALUES OF THE FUNDAMENTAL PHYSICAL CONSTANTS

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Ways of increasing the accuracy of the values of the Planck and Avogadro constants in experiments with Watt balances and crystalline silicon spheres are examined. These are needed for the new definitions of the kilogram and mole. The advantages and disadvantages of fixing the values of a number of fundamental physical constants when introducing definitions of the SI units are discussed, in particular a new definition of the unit of mass based on a fixed value of the Avogadro constant.
Key words: *measurement standards, dimension of the unit of mass, fundamental physical constants, Avogadro number, Planck constant.*

The standard for the unit of mass is the only standard for the International System (SI) fully defined by an artificially created international platinum-iridium prototype *K* for the kilogram. Another three fundamental units of the SI, the ampere, mole, and candela, are specified indirectly with the aid of the kilogram, so any imperfection in the prototype *K* also affects their definitions. Thus, the ampere is defined in terms of the force of the interaction between two conductors with a current. The candela, the unit of luminosity, is defined in terms of the unit of power, where force and power are related to the unit of mass. The unit of the amount of matter, the mole, is defined in terms of the unit of mass as the amount of a substance containing as many units of this substance as atoms of the carbon isotope ^{12}C are contained in a mass of 0.012 kg [1].

It is known that uncontrolled variations in the magnitude of *K* relative to its copies caused by contamination and wear are as much as $5 \cdot 10^{-8}$ kg over the 100 years to 1990. A similar time drift for the entire system of standards is also entirely possible. The international prototype for the kilogram is available only in one place, so it is difficult to transfer it to working measurement sites; it might be damaged or even destroyed. The high density of the platinum-iridium alloy of which the copies of K are made leads to a reduction in the accuracy of comparisons in air with the secondary kilogram standards of steel that are used in practice in further transfers of its scale or in making measurements. The minimum standard uncertainty in practical measurements which is usually attained with the existing method for transfer of the unit of mass is at a level of 10^{-7}. This uncertainty cannot be reduced significantly in the future, so this will limit any increase in the accuracy of measuring a number of other quantities, besides mass. Thus, there is a pressing need to redefine the existing standard *K* and the question of redefining a number of other fundamental SI units, such as the ampere, kelvin, and mole arises [2–5].

The numerical value of a physical quantity depends on its size and the units of measurement that are used. The uncertainty with which the quantity can be measured also depends on the chosen unit of measurement. For example, the relative uncertainties in measurements of the mass of the electron, proton, neutron, and α-particle in kilograms are at a level of 10^{-7}, while in atomic units (amu) they are at a level of 10^{-10}. The proposed conversion to new definitions of a number of SI units based on the fundamental constants of physics (FCP) [6, 7] will provide higher accuracy, stability, and reproducibility

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of measurement results. An example is the conversion to an atomic standard for the units of length and time (frequency), which would permit a considerable increase in the accuracy of reproducing these units and in the accuracy with which the speed of light is determined. Then, by fixing the value of the speed of light, it would be possible to introduce a new standard for the unit of length, the so called light meter. Here the units of quantities that are defined with the aid of the FCP become independent, in principle, of many of factors associated with the interaction of natural conditions and human activity.

Although the basic units chosen for the SI are not the same as the FCP, given the unity of nature, it is always possible to represent them as derivative units of these constants. For example, the effective units of length and time depend on the constants of quantum electrodynamics – the speed of light c, the electronic charge e, electronic mass m_e, and Planck constant $\hbar$ [8]. Thus, the problem of defining the system of basic units actually reduces to choosing the required set of FCP. This problem does not have a unique solution, since various sets of these constants can be proposed to solve it. The sets of FCP and basic units are naturally interrelated, and, in theory, increasing the number of basic units of measurement leads to an increase in the number of constants [9].

In 2005, it was proposed [2] that the redefinition of the unit of mass not be delayed and that this should be done on the basis of fixing either the Planck constant or the Avogadro constant. In that same year, the International Commission on Weights and Measures (CIPM) accepted a recommendation on preparatory measures toward the introduction of new definitions of the SI units: kilogram, ampere, kelvin, and mole [4]. It was proposed that this be done relying on the following assumptions: the general structure of the SI – the basic quantities and their units – should remain unchanged, since it satisfies the current and will satisfy the future requirements of metrologists and the entire scientific community. It is not obligatory that a new definition of an SI unit ensure a reduction in the uncertainty of its implementation. For metrology and for science as a whole, the advantages of replacing the existing definition of the kilogram with a definition associated with an exact value of the Planck constant $\hbar$ or the existing definition of the kelvin by a definition associated with an exact value of the Boltzmann constant k_B may be of greater significance than any slight increase in the uncertainty that might be possible with such a replacement of the unit of mass or thermodynamic temperature. In addition, a new definition of any unit must not lead to a discontinuity in the values of this unit. This means that the chosen values of the Planck constant $\hbar$, the elementary electrical charge e, the Boltzmann constant k_B, and the Avogadro constant N_A used in the new definitions must be so close to their values in the existing SI as modern means of measurement make possible.

Based on these assumptions, several variants of new definitions for these four basic units were proposed [3]. These definitions involve fixing the values of four FCP: the Planck constant $\hbar$, the elementary electrical charge e, the Boltzmann constant k_B, and the Avogadro number N_A, in addition to the previously fixed value of the speed of light c. In the following, we examine the major versions for a new definition of the unit of mass [2–5, 10] and, in more detail, the version of such a definition based on a fixed value of the Avogadro number. Some advantages and disadvantages of fixing the values of the FCP are pointed out.

The Major Methods for Implementing a New Definition of the Unit of Mass. Strictly speaking, to ensure compatibility with the existing standard and previous mass measurements, the relative uncertainty in transferring the unit of mass should be at a level of 10^{-9} and the instability of the unit should be less than $5 \cdot 10^{-10}$ kg over a year. Two major possible new definitions of the unit of mass have been proposed, specifically: using the values of the Planck constant $\hbar$ or the Avogadro constant N_A, which, however, are presently determined with insufficient accuracy (on the order of $2 \cdot 10^{7}$) for this purpose. According to the resolutions of the committees of the CIPM and the 23rd General Conference on Weights and Measures (CGPM) the relative standard uncertainty in the values of $\hbar$ and N_A required for replacement of the international prototype for the kilogram should be no more than $2 \cdot 10^{-8}$ [4, 5]. We shall analyze the two major methods for implementing a new definition of the unit of mass, which, when the existing prototype K is used, are also methods for determining the Planck and Avogadro constants.

In the method of defining the Planck constant with a Watt balance, mechanical and electrical powers are compared. The idea behind this method is the following [11]: let us consider a frame with a current I and a side of length l that can be moved vertically in a uniform magnetic field **B** perpendicular to the plane of the frame. The force acting on the moveable side is

$$F = BIl = mg,$$

where m is the mass balancing this force, and g is the acceleration of gravity. If the same side of the frame moves at a velocity u perpendicular to **B** and l without a current, then a voltage $U = Blu$ appears at its ends. Eliminating the product Bl, we

TABLE 1. Budget of the Major Systematic Relative Standard Measurement Uncertainties in a Determination of the Planck Constant

Sources of uncertainty	Relative standard uncertainty, nW/W	
	2005	2006
Voltage, current	12	15
Mass	15	10
Resistance	10	10
Acceleration of gravity	30	12
Profile of laser wave front	10	10
Suspension wheel	20	2
Magnetic field perturbations	11	7
Mathematical signal processing	16	16

obtain $UI = mgu$. The Josephson effect and the quantum Hall effect are used to measure the voltage U and the current I, so it is possible to rewrite the above equation with practical electrical units and fixed values of the Josephson constant $K_{\text{J-90}}$ and von Klitzing constant $R_{\text{K-90}}$ [12]:

$$\frac{mgu}{\{U^2 / R\}_{90} K_{\text{J-90}}^2 R_{\text{K-90}}} = \frac{\hbar}{4}. \quad (1)$$

In order to use Eq. (1) for defining the unit of mass, it is necessary to fix the Planck constant $\hbar$. Thus, it is possible to create a so-called electrical kilogram using a Watt balance.

The main source of the measurement uncertainty by this method is humidity and the limits on the accuracy of the measurements of voltage, velocity, and mass. These effects can be minimized by placing the Watt balance in a vacuum. Given the conceptual layout of Watt balances and the possible sources of noise, the accuracy with which the size of the unit of mass can be transferred using the electrical kilogram is most likely to remain at a level of 10^8 [10]. The main sources of uncertainty in determining h with a Watt balance at NIST (USA) during 2005–2006 [13] are listed in Table 1.

It is necessary to reduce the influence of all or some of these factors in order to bring the combined relative standard uncertainty of the result to 20 nW/W by 2010.

Recently, an experiment was completed on the determination of the Planck constant using a Watt balance at the NPL (Britain). The structure of the sources of uncertainties in the final result is basically the same as before: adjustment, humidity, mass measurement, ratio of voltage to velocity. The relative difference in the results from NPL and NIST was about 0.3 ppm, with the NPL result being higher and closer to the result obtained in the Avogadro project [13].

It is possible that there are unaccounted systematic uncertainties in the NPL and NIST experiments. For example, in analyzing the experimental data it is always assumed that $K_J^2 R_K = 4/\hbar$. However, this relationship has not yet been proved rigorously, so it is necessary to use the more general relation

$$K_J^2 R_K (1 - 2\varepsilon_J - \varepsilon_K) = 4/\hbar, \quad (2)$$

where ε_J and ε_K are the uncertainties accounting for the possible derivations of K_J and R_K from $2e/\hbar$ and $\hbar/e^2$, respectively.

Then Eq. (1) must be modified to

$$\frac{mgu(1 + 2\varepsilon_J + \varepsilon_K)}{\{U^2 / R\}_{90} K_{\text{J-90}}^2 R_{\text{K-90}}} = \frac{\hbar}{4}. \quad (3)$$

Another source of unevaluated systematic uncertainties may be the measures employed to suppress beating in the ratio U/u in phase with the motion of the frame through the magnetic flux.

The method of replacing the kilogram prototype K by the Avogadro constant and the atomic mass unit offers some promise [2, 3, 5, 10, 14, 15]. Given that the relative uncertainty attainable in comparisons of the masses of atoms, ions, electrons, and protons in a.m.u. is at a level of 10^{-10}, the main difficulty lies in counting the number of atoms (molecules) with the required accuracy. For this, experimental measurements are used to find the mass m and sample volume V, after which the density $\rho = m/V$ of the sample and its average molecular mass $M = m/n$ are found, where m is measured in kilograms, and n is the amount of matter in moles. At the same time, it is possible to measure the microscopic volume v of a lattice cell by XRCD (x-ray crystal diffraction) based on a set of data from an x-ray interferometric study of the crystal lattice parameters. This yields the volume $v = a_0^3$ of a unit cell of a silicon crystal lattice, the number f of atoms in a unit cell, and the microscopic volume $v_a = v/f$ of an atom. As a result, the molar volume can be calculated in two ways:

$$V_M = M/\rho; \qquad V_M = N_A v/f. \tag{4}$$

Equating the two parts of Eq. (4), we obtain the Avogadro number:

$$N_A = Mf/(\rho v). \tag{5}$$

We now list the steps required to determine the Avogadro constant using samples of crystal silicon: producing a silicon sample; determining the average molar mass M; growing a single crystal sample; mechanical working and producing silicon spheres; polishing the spheres and monitoring the spherical surface; determining the parameters of the crystal lattice by XRCD; determining and taking account of corrections (impurities, lattice defects, etc.); and calculating N_A.

Each stage of this effort contributes to the total budget of uncertainty for the measurement results. The main contributions arise in determining the isotopic composition of the silicon, which is composed of three stable isotopes, ^{28}Si, ^{29}Si, and ^{30}Si, the lattice crystal parameter, and the volume of a sphere. Counting the silicon atoms assumes information on the number and types of impurities in the silicon, as well as on the effect of impurities on the mass and volume of the sample, since, no matter how carefully the silicon crystal is grown, it may contain thin layers of amorphous silicon, silicon dioxide, and other impurities. The possible presence of various kinds of crystal lattice defects must also be taken into account. All these defects in a sample will affect the accuracy with which its macroscopic density is determined, while the surface layer, which contains the largest amount of impurities, will contribute to an apparent displacement of the surface during interferometric measurements of the sample diameter.

A large loss of accuracy occurs during the determination of the molar mass $M(\mathrm{Si})$ of the samples, which can be expressed in terms of the molar mass of one of its isotopes as $M(\mathrm{Si}) = M(^{28}\mathrm{Si})(1 + \xi)$, where ξ must be measured with an uncertainty of $1\cdot10^{-8}$. This kind of measurement is extremely difficult and reduces to a determination, for example, of the isotopic composition of a gaseous mixture of SiF_4 ions with a mass spectrometer. An experiment in 2003 [16] with a natural mixture of the isotopes ^{28}Si, ^{29}Si, and ^{30}Si yielded the following value for the Avogadro number, with a relative standard uncertainty of $3.1\cdot10^{-7}$:

$$N_A = 6.0221353(18)\cdot10^{23}. \tag{6}$$

A second stage of this experiment is now under way. In order to reduce the contributions from uncertainties associated with the determination of the molar mass, samples enriched in ^{28}Si to 99.985% are being used [17]. The uncertainties have been evaluated at NMIJ (Japan) [14]. The relative standard uncertainties in the determination of the Avogadro constant obtained in the first stage of the experiment and their proposed values in the final stage of the experiment in 2010 are listed in Table 2.

Comparison of the Two Basic Methods for Redefinition of the Unit of Mass Associated with the Avogadro and Planck Constants. The method based on the Avogadro constant, where the density of crystalline silicon is determined radiographically, is referred to as the silicon sphere (ball) method. The method based on an exact value of the Planck constant,

TABLE 2. Budget of Relative Standard Measurement Uncertainties in a Determination of the Avogadro Constant

Sources of uncertainty	Relative standard uncertainty	
	obtained	proposed
Molar mass	$2.0 \cdot 10^{-8}$	$1.0 \cdot 10^{-8}$
Volume	$3.0 \cdot 10^{-8}$	$1.0 \cdot 10^{-8}$
Lattice constant	$1.8 \cdot 10^{-8}$	$1.0 \cdot 10^{-8}$
Impurities, lattice defects	$2.0 \cdot 10^{-8}$	$1.0 \cdot 10^{-8}$
Surface quality	$1.0 \cdot 10^{-8}$	$0.5 \cdot 10^{-8}$
Mass	$0.5 \cdot 10^{-8}$	$0.5 \cdot 10^{-8}$
Calculating N_A	$4.6 \cdot 10^{-8}$	$2.1 \cdot 10^{-8}$

where electrical and mechanical powers are compared, has come to be known as the Watt balance method. If the Planck constant is specified, then this method can be used to implement and reproduce the unit of mass. The two methods are related [2, 3, 15] since there is a formula which includes other FCP that are known to better accuracy (on the order of 10^9) than the values of $\hbar$ and N_A:

$$\frac{R_\infty}{A_r(e)\alpha^2} = \frac{M_u c}{2\hbar N_A}, \tag{7}$$

where R_∞ is the Rydberg constant; $A_r(e)$ is the relative atomic mass of the electron; α is the fine structure constant; M_u is the molecular constant, equal to 10^{-3} kg/mole with $M_u = M(^{12}C)/12$ and $M(^{12}C)$ as the molar mass of carbon; and c is the speed of light in vacuum. The last two constants (c, M_u) are defined exactly, while the first three ($A_r(e)$, R_∞, α) are determined experimentally but with much greater accuracy than N_A and $\hbar$. The relative difference in the values of the Planck constant obtained by two methods is 10^{-6}. Since the result is proportional to the Planck or Avogadro constant, respectively, when the kilogram is reproduced by the Watt balance or silicon sphere method, further improvements are need in these methods in order to achieve the required accuracy and mutual consistency.

At present, in order to avoid the discrepancy in the values, it is customary to take N_A from Eq. (7), with $\hbar$ determined in experiments with a Watt balance. With relative standard uncertainties of $5.0 \cdot 10^{-8}$, the current values of the Planck and Avogadro constants are [18, 19]

$$\hbar = 6.62606896(33) \cdot 10^{-34} \text{ J}\cdot\text{sec} \tag{8}$$

and

$$N_A = 6.02214179(30) \cdot 10^{23} \text{ mole}^{-1}. \tag{9}$$

However, using the Watt balance with subsequent introduction of an electrical kilogram based on the electrical quantum units of resistance and voltage will require checking the mutual consistency of the currently recommended values of the Josephson and von Klitzing constants by means, for example, of apparatus based on single electron tunneling; that is, it will actually require the creation of a quantum-mechanical current standard. Then it becomes possible to close the so-called quantum triangle and to establish a consistent system of practical electrical units [20–22]. This, in particular, will be a test of the relationship $K_J^2 R_K = 4/\hbar$ and require inclusion of the corrections ε_J and ε_K in Eq. (2).

The two methods discussed here have their advantages and disadvantages [2, 3, 5, 10]. For example, the relative standard uncertainty (u_r) in the determination of the electronic charge with the unit of mass determined by the second method will be three times smaller than the uncertainty obtained using the first method, and u_r for the nuclear and Bohr magnetons, by a factor of one and a half. The relative standard uncertainties in the determination of the masses of the elementary particles are considerably smaller when the unit of mass is defined by the first method. In this case, u_r for the electron mass is fifteen times smaller and for the proton mass, fifty times. Here the atomic unit of mass will be obtained with zero standard uncertainty. Thus, from the standpoint of the accuracy of determining the masses of the elementary particles and, therefore, of the accuracy of theoretical calculations of the masses of more complicated systems of particles, including evaluating the masses of macroscopic objects, in this case the method based on the Avogadro number and the atomic unit of mass is to be preferred. This method is logically consistent and intuitive, which also makes it convenient for use in education. In addition, this method can be used for a simultaneous definition of the unit of the amount of matter. The following is a detailed analysis of one of the possible ways of introducing a new definition of the unit of mass using a fixed value of the Avogadro constant.

Introducing New Definitions of the Fundamental SI Units Using Fixed Values of the FCP. We now examine the procedure for fixing the values of the FCP for defining the fundamental SI units, its advantages, and, also, its disadvantages. The proposal for redefinition of the four units on the basis of fixed values of the FCP is based on the same principle used for the new definition of the meter in 1983. It is proposed that the four constants $\hbar$, e, k_B, and N_A be fixed as exact values, known with zero uncertainty, and that they be used to define the kilogram, ampere, kelvin, and mole. However, in introducing the definition of the light meter in 1983, conditions for the high precision determination of the frequency and velocity of light were attained which were roughly two orders of magnitude greater than the accuracy of the length measurement. This situation made it possible to fix the value of the speed of light. As for the above four constants, $\hbar$, e, k_B, and N_A, these conditions have not yet been met, so that premature fixing of the current values of all these constants might interfere with the development of both basic physics and metrology.

In fact, on a practical level, a set of units of fundamental physical quantities should be determined that is necessary for support of measurements of physical quantities with a specified precision. Here the definitions of units for the practical system, i.e., those which are reproduced in real measurement systems, must be operational. Thus, although in theory the errors in reproducing the "light meter" are caused by errors in reproducing the unit of frequency (time), in practice the measurement errors for lengths depend on errors in measuring the positions of interference fringes of laser light. Hence, in practice it is preferable to define the meter in terms of the wavelength of coherent laser light, rather than in terms of the speed of light, while in a fundamental system of units it is preferable specifically to fix the speed of light. A similar situation arises with the proposed new definitions of other SI units. Here the procedures for comparing practical and fundamental units will have to be developed in detail.

At present, the modern tendency to develop the SI with fixed values of a series of FCP and define the units in terms of these values involves a "movement" of fundamental metrology toward the natural units employed in physics and based on certain sets of FCP. Fixing the values of the fundamental constants c, $\hbar$, e, k_B, and N_A offers the possibility of defining a series of basic SI units: meter, kilogram, ampere, kelvin, and mole. In the author's opinion, the number of FCP in any such set which are to be fixed must be reduced to a minimum. The proposed fixing of a large number of constants, as recommended, for example, by some committees of the CIPM, originates in a desire to solve several problems at once: to introduce a new definition of the unit of mass and to legitimize the use of definitions of electrical units on the basis of the quantum mechanical Hall and Josephson effects. However, at present there is no theory for these effects which might be used, for example, to calculate the correction factors for the Josephson and von Klitzing constants as functions of sample shape or the conditions in an external medium. The introduction of a fixed value of the Boltzmann constant also does not originate in practical needs. It is known that the existing precision of thermodynamic temperature measurements is sufficient for practical measurements and cannot be significantly increased in the foreseeable future [23]. Given this, we shall assume that only the introduction of new definitions for the units of mass and of the amount of a substance based on a fixed value of the Avogadro constant N_A^* is justified. (In the following, an asterisk denotes units defined with this fixed value for the Avogadro number.)

We consider the following definition of the unit of mass as fundamental: *the unit of mass, the kilogram, is the exact mass of* $\{N_A^*\}/0.012$ *free atoms of* ^{12}C *carbon atoms in a state of rest and in their quantum mechanical ground state.*

The great advantage of the proposed definition is that it simultaneously introduces a new definition of the amount of a substance that is consistent with the current one, specifically: *the unit of the amount of a substance, the mole, contains* $\{N_A^*\}$ *structural units of the given substance.*

As this definition implies, the value of $\{N_A^*\}$ must be a multiple of 12 [15]. In that case, the kilogram and gram will contain an integral number of atoms of carbon ^{12}C atoms. This is entirely natural and should be satisfied. Thus, a natural invariant, the mass of the ^{12}C carbon atom, is the basis of the new definition of the kilogram. The major problem is choosing a value of $\{N_A^*\}$ within an allowable interval such that the relative difference between the values of the existing kilogram $\{K\}$ and the new kilogram $\{K^*\}$ should be on the order of 10^{-8}. In this case, it is possible to retain the existing value for the molar mass of carbon $M(^{12}C)$ = 12 g/mole and the molecular mass constant M_u =1 g/mole [24]. It is also desirable that the new molar mass and molecular mass constant should be expressed in terms of the kilogram (kg*) and mole (mole*) in the following way:

$$M(^{12}C) = 12 \cdot 10^{-3}\ \text{kg}^*/\text{mole}^*, \quad M_u = 10^{-3}\ \text{kg}^*/\text{mole}^*. \tag{10}$$

This version of the new definitions for the kilogram, mole, molar mass, and molar mass constant is very convenient, since it does not violate the existing practice for calculating mass and molar mass in chemistry.

As an example, let us see how it might be possible to fix the value of the Avogadro number $\{N_A^*\}$. Of course, it is of prime importance that the values of N_A obtained in silicon sphere experiments (6), which lie within the range

$$N_A = (6.0221335\text{–}6.0221371) \cdot 10^{23}, \tag{11}$$

be taken into account.

The boundaries of this interval will be more exactly known after the completion of the Avogadro project in 2010. Then it is possible to use the condition proposed in Ref. 25: starting with the structure (diamond-like) of the silicon crystalline lattice it was calculated that, in a three dimensional sample with a cubic shape with an equal and integer number k of lattice elements along each edge of the cube, the number of atoms should be $N = 6k^3 + 8k^2 + 15k + 4$. In order for the number N to fall within the allowable interval (11), it is necessary to choose $k = k^*$. If the resulting number N^* is not divisible by 12, then it is possible to use the fact that a physical, rather than mathematical, constant is being defined. In this case, varying the number of atoms in the sample by a small number n on the order of 10 will lie beyond the limits of experimental accuracy, both now and in the foreseeable future. Thus, to N^* we add another n atoms and obtain a number $N^* + n$, which is divisible by 12. It can be chosen as the fixed value of the Avogadro constant: $\{N_A^*\} = N^* + n$.

This new definition of the unit of mass based on a fixed value of $\{N_A^*\}$ maintains a succession with the current definitions of the kilogram and mole and, when it is introduced, it will not disrupt the existing metrological chains of transfer for the sizes of the units of mass and of the amount of a substance, or existing practice for the measurement of mass and molar mass.

At present, the predominant opinion holds that a new definition of the kilogram should be based on a fixed value of the Planck constant $\hbar$, the fundamental constant of quantum mechanics, as c is the fundamental constant of the theory of relativity. In addition, a redefinition of the ampere through fixing the electronic charge e, when the values of $2e/\hbar$ and $\hbar/e^2$ have precisely determined values will provide some advantages for the metrological support of electrical measurements and will transfer the practical electrical units Ω_{90} and V_{90} into the ranks of the SI units. However, according to the quantum theory of interactions, the electrical charge is not a constant. Its value depends on the energy of interacting particles [26], so it is better to use a fixed value of the vacuum magnetic permeability (magnetic constant) μ_0. The Planck constant can also appear to depend on the energy and space-time scales of processes under extreme conditions [27]. At the same time, a fixed value of the Avogadro constant can be equated to an absolute physical constant under any conditions.

Conclusion. The possibility of replacing the existing definitions of the four basic SI units, the kilogram, ampere, kelvin, and mole, with definitions based on exact values for the Planck, Avogadro, and Boltzmann constants and the electronic charge has been under intense study in recent years. Here the problem of raising the precision with which these constants are determined is fundamental to a conversion to new definitions for these units. Solving it will require reducing the systematic uncertainties, among which a major contribution is from the uncertainties associated with aligning the experimental apparatus and creating the required samples. Thus, there should be more experiments to determine the exact values of the FCP.

In accordance with the recommendations of the committees of the CIPM, as well as the resolutions of the 23rd CGPM, it is necessary to prepare the transition to new definitions of the kilogram, ampere, kelvin, and mole that will be based on exact values of these constants. In this regard, national metrological institutes are to conduct appropriate experiments so that it will be possible to adopt a well based solution for the redefinitions of these four fundamental SI units using fixed values of the FCP. They are also to develop practical means for implementation of any new definitions based on fixed values of the constants. Given the lack of consistency among the values of the uncertainties in the results obtained by two different methods for redefining the kilogram and mole, the Consultative Committee on units has recommended to the 23rd CGPM that it defer any new definition of the kilogram until the current inconsistency between the results for $\hbar$ obtained with the Watt balance and silicon sphere methods is resolved.

The results of this paper raise doubt as to the need to fix the values of the Planck constant and the electronic charge for new definitions of the basic SI units. On one hand, if a new definition of the kilogram with a fixed value of $\hbar$ is chosen, the basis of this definition will not be a natural invariant, such as the mass of a carbon atom, but an artificially created electromechanical device, the Watt balance, with a large number of sources of systematic uncertainty. On the other hand, if the Avogadro number is fixed, an exact method for measuring the Planck constant becomes available using values of the electronic mass, Rydberg constant, and fine structure constant. Here the Watt balance becomes a method for exact determination of the product $K_J^2 R_K$, which will facilitate the development of a theory of the macroscopic quantum mechanical Josephson and Hall effects.

Thus, we propose that the new definitions of the units of mass and of the amount of a substance should be based on a fixed value of the Avogadro constant and the mass of the carbon atom [5, 10]. The Avogadro number can be fixed, for example, in the way described above. That method leads to intuitive and logically sound definitions of the kilogram and mole that are compatible with the existing definitions of these two units and with existing practice for measurements of mass, molecular mass, and the amount of a substance.

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