The SI's Base Quantities and Units

W. H. Emerson

Abstract *The author seeks to demonstrate that the much-used, much quoted* International System of Units *rests on shaky foundations. The current proposals for its revision do nothing to correct that. These foundations derive from the I*nternational System of Quantities, *which is not undergoing revision.*

The International System of Units (SI8) is founded on seven base quantities, for which it defines seven base units.. Most of them are either not quantities at all, as that concept is defined internationally, or they do not satisfy the definition of base quantities.

Definitions of the concepts *quantity, unit, base quantity* and *base unit* are given in the 3rd edition of the International Vocabulary of Metrology (abbreviated to VIM3 from its French title). Names for base quantities, but not always definitions, are published in the standard ISO 80000 of 2009 by the International Organization for Standards (ISQ). Units are defined in the International System of Units and published in the SI Brochure 8th edition (SI8}. Logically the definitions of the terms *quantity* and *unit* given in the International Vocabulary must apply where those terms are used in the other international documents. Close examination shows that those definitions are frequently disregarded,

Countable 'quantities'

According to VIM3, (1.4 NOTE 3 'Number of entities can be regarded as a base quantity in any system of quantities': and again in 1.10 Note 3. According to the SI Brochure 8th edition Sction 2.2.3, "Another class of dimensionless quantities are numbers that represent a count, such as a number of molecules …, All of these counting quantities are also described as being dimensionless, or of dimension one, and are taken to have the SI unit one, although the unit of counting quantities cannot be described as a derived unit expressed in terms of the base units of the SI. For such quantities, the unit one may instead be regarded as a further base unit." These assertions are made as Notes appended to the definitions. They do not carry the same weight as the definitions themselves and may even contradict them.

The orthodox view appears to be that entities defined as of a particular set of entities may be counted, and the result of such a counting is a quantity of a kind known as 'number of entitles' of the specified kind. Its unit is either 1 or one entity. But an entity is not a quantity according to VIM3's definitions of quantity and of unit. It is not a *property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference* (VIM3 1.1).[i]

Amount of substance

The SI unit *mole* is declared a unit of *amount of substance,* and indeed that term appears in the definition of the unit. Amount of substance is a relatively new term and the quantity is denominated a base quantity in the ISQ and the SI. The current definition of the unit of amount of substance, the mole, provided by the SI [SI8] reads:

The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is .mol. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.

Unless the undefined .'amount of substance' comprises something in addition to the specified elementary entities (there has been no suggestion that it does) a mole of carbon-12 atoms is also the number of atoms in 0.012 kg of carbon 12, and a mole of any kind of elementary entity is a number of such entities equal to that same number of atoms of carbon 12.

A definition of mole, consistent with the SI definition and without reference to an undefined quantity may therefore be stated thus:

A mole is a number of elementary entities equal to the number of atoms in 0.012 kg of carbon 12.

That is a definition of a 'unit' of amount of substance, so a definition of amount of substance follows:

An amount of substance is a number of elementary entities of a specified kind.
Its SI unit is the mole of the specified substance.

The SI's definition makes clear what is meant by an 'elementary entity'. In a mole of such entities; all of them are identical and of the specified kind. None of them is a '*property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference* (VIM3 1.1) It is what is sometimes called a countable quantity and by that token it is said in a Note in VIM3 to be a base quantity. The VIM offers no argument or authority for that assertion. The entities which one counts may have properties which are quantities, but the entities themselves are not quantities (see above under **countable 'quantities')**. Amounts of substance are not quantities and therefore not base quantities. Moles are not base units.

Electric current

Physically the quantity electric current accompanies, in the negative sense, a rate of flow of electrons in a conductor or thermionic device, or of ions in an electrolyte, or of holes in a semiconductor. It is a flow rate of elementary charges. The International Unions of Pure and Applied Chemistry (IUPAC) and of Physics (IUPAP) define it [4] as

$$I = \mathrm{d}C/\mathrm{d}t, \quad (5)$$

where *C* is a "quantity of electricity". and *t* is a time, making electric current, in this (non-ISQ) system of quantities, a derived quantity. *C* is an electric charge, physically a number of elementary charges and thus, in principle, countable. The reference does not define the concept .elementary charge; nor is the term used

.

However, electricians do not count the rate of passage of electrons. An electric current is expressed as a number and a unit that is not an elementary charge per unit of time, and, in the SI, the unit is the ampere. It is defined as:

> *that electric current which, if maintained in two straight, parallel conductors of infinite length, of negligible circular cross-section, and placed one metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.*

It could logically be defined as:

> *time derivative of the number of elementary charges traversing a surface and equal to that constant rate of passage of such charges which, if maintained in two straight, parallel conductors of infinite length etc..*

Thus, when the ampere is realised, electricians can obtain a value of a current that can be compared with any other, without their having any knowledge of the number of electrons passing a section in a conductor in a given time. The ampere serves electricians in a way similar to that in which the mole serves chemists.

It is now being proposed, in the context of a proposed redefinition of the kilogram [5], that the ampere be redefined as a flow of a fixed number of elementary charges per second. In an earlier draft of this essay I wrote that a 'number of elementary charges' is of the same nature as 'number of elementary entities' and so is not a quantity.[ii] Ingvar Johansson pointed out to me that an elementary charge is not an electron, which is an elementary entity, but a property of an electron and of other carriers of elementary charges. Electric current can rightly be a derived quantity of the SI, and its unit, ampere, maybe defined as a fixed number of such charges per second. It is not a base quantity.

Thermodynamic temperature

The concept of temperature, of hotness or coldness felt through the skin, is known to all of us. But our skin sensors are unreliable in quantifying differences of temperature, so we resort to the observation of some measurable effects of changes of temperature, such as changes in the volume of a fluid in thermal equilibrium with the medium whose temperature is of interest, as typified by a liquid-in-glass thermometer. We can put marks on the thermometer where it is in equilibrium with each of two media in turn, each of fixed and constant temperature (melting ice, boiling water at atmospheric pressure) and construct a linear scale of temperature between them. Thermometers also allow us to extend the range of measurable temperature well beyond that which skin can tolerate.

The concept of *thermodynamic* temperature was first defined by Lord Kelvin and Sadi Carnot. The efficiency of any reversible heat engine is that of a Carnot cycle which receives a quantity of heat q_1 reversibly at constant temperature T_1 and rejects the quantity q_2 reversibly at constant temperature T_2. It is equal to

$$(q_1 - q_2) / q_1 = 1 - q_2 / q_1.$$

A thermodynamic temperature T is defined such that $T_2 / T_1 = q_2 / q_1$. The efficiency of a

Carnot engine is a function only of thermodynamic temperatures and is independent of the working medium. The maximum possible efficiency occurs when $T_2 = 0$, so 0 is the lowest possible value of temperature T_2, or of any thermodynamic temperature. Thermodynamic temperature, formerly known as .absolute temperature, has an absolute zero. A reversible engine is one that receives and rejects heat infinitely slowly and so it produces an infinitesimal amount of power.

For anyone unimpressed by engines which produce virtually no power, but which do it with predictable efficiencies, there are other definitions of thermodynamic temperature. The classical concept of an ideal gas is that of one in which the molecules occupy a volume that is infinitesable in relation to the volume of the container, do not interact, and collide elastically with the walls of the container. Such gases are the subject of the Ideal Gas Law, derived from Boyle's Law and Charles' Law:

$$PV = nRT = NkT$$

where P is pressure, V is volume, n is amount of gas, R is called the universal gas constant, N is the number of atoms or molecules, k is called the Boltzmann constant and T is thermodynamic temperature. nR and Nk are the same quantity presented in different ways because of the different units used to express the quantity of gas. They are the unfortunate result of involving units in the definitions of quantities. kT *is* the energy per molecule; RT is the energy per unit of amount of gas: an arbitrarily defined number of molecules. R and k are called constants because they are independent of the chemical nature of the gas.

The SI defines a scale of thermodynamic temperature, the Kelvin scale, wherein the temperature at the triple point of water is assigned the number 273.16. A difference of one on the Kelvin scale is one 273.16th of the temperature of the triple point of water. The fixed fundamental temperature on the Kelvin scale is expressed in the SI as 273.16 kelvin, abbreviated to 273.16 K.

A thermodynamic temperature is always and essentially in relation to absolute zero as datum. No thermodynamic temperature can be added meaningfully to any other thermodynamic temperature. The quantity $T_3 = T_1 + T_2$ is unknown to science. A temperature of one kelvin, that is of 1 on the Kelvin scale, is a particular thermodynamic temperature. A temperature of three on that scale cannot be 1 kelvin + 1 kelvin + 1 kelvin, the sum of three thermodynamic temperatures; it is a temperature of 3 on the Kelvin scale. To be consistent with the definitions of thermodynamic temperature alluded to above the expression 1 kelvin cannot denote a unit of thermodynamic temperature.

However, a difference of 1 on the Kelvin scale can usefully be called a unit of temperature difference. Such a quantity has no absolute zero; it can be positive or negative. Differences of thermodynamic temperature can be added or subtracted. A value of a thermodynamic temperature used formerly to be expressed as so many degrees Kelvin; a value of difference of temperature was given as so many Kelvin degrees, thus making the distinction. A *unit* of thermodynamic temperature *difference* should not be confused with a thermodynamic temperature. It would be logical to use the name kelvin (or some other) for the unit and indicate a thermodynamic temperature n on the Kelvin scale by calling it n Kelvin.

The matter is especially of interest in relation to the definition of a 'unit' of entropy. The quantity is defined by the equation

$$dS = dQ / T,$$

where *S* is entropy and *Q* is energy. Thus if the energy of a system changes by one unit of energy at constant temperature *T*, its entropy changes by that unit divided by *T*. If we put

$$dS = C dT / T,$$

where *C* is heat capacity, we cannot say that a unit of d*T* (a difference of temperature) is also a 'unit' of *T* (a thermodynamic temperature and having no unit) and conclude that entropy has a unit equal to that of heat capacity. Entropy has no unit that is independent of thermodynamic temperature.

The above view is, however, at variance with that of the CGPM, which has declared (quoted in SI8) that *the unit of thermodynamic temperature is denoted by the name Kelvin, and its symbol is* K*; the same name and symbol are used to express a temperature interval; a temperature interval may also be expressed in degrees Celsius*. (13th Meeting 1967-1968, Resolution 3). The SI Brochure adds, "*...it remains common practice to express thermodynamic temperature, symbol T, in terms of its difference from the reference temperature* T_0 *+273.15* K*, the ice point*.. That difference is a Celsius temperature, so the Brochure effectively states that thermodynamic temperature may be expressed in terms of Celsius temperature. At the same 13th Meeting, Resolution 6, the CGPM declared the unit of entropy to be the same as that of heat capacity.

None of this has much, if any, relevance to the *measurement* of temperature. The current definition of the Kelvin Scale has one realisable fixed point: the triple point of water, and one that is not realisable: zero. All other temperatures remain undefined. Or rather they are defined theoretically by the properties of non-existent ideal gases to interpolate between the fixed points, and practically by the International Temperature Scale ITS-90. The latter has eighteen fixed points and complicated empirical equations to interpolate between them. ITS–90 is not a scale at all; it is an *equipment calibration standard*. All measurements of temperature are made with reference to that standard and are generally expressed in 'degrees Kelvin' with no reference to the SI definitions.

The SI's equating thermodynamic temperature to interval of temperature has serious implications for heat capacity and entropy, to which the SI assigns the same unit. Heat capacity is defined as

$$c = dQ/dT,$$

with the unit joule per kelvin. Entropy is defined by the equation

$$dS = dQ/T.$$

The SI gives it the same unit as for heat capacity, at the same time acknowledging that the quantities are of different kinds. Where they differ is in d*T* and *T*. The former is a difference of temperature; the latter is an absolute temperature, which has no unit; it has a number on a scale. Tables of thermodynamic properties express entropies by integration from an arbitrary datum thermodynamic temperature. They are much used by engineers who understand the very practical difference between absolute temperature and temperature difference.

In the proposed New SI the triple point of water is abandoned as the fixed point on the Kelvin Scale in favour of a fixed (exact) value of the Boltzmann constant, expressed as $1.380\ 65 \times 10^{-23}$ joule per kelvin (JK^{-1}). Here again we find the SI using kelvin as though it were a unit of absolute temperature, and the same objections apply.

Thermodynamic temperature is not a quantity whose magnitude can be expressed by the product of a number and a unit. It cannot be a base quantity of the ISQ. kelvin cannot be a unit of the SI.

Luminous intensity

The concept of luminous intensity is of the intensity of light *perceived* by a human eye, as opposed to that actually *received* by the eye. It is the intensity of light visible to that eye. That is not, however, a concept that is consistent with the SI unit, the candela. A physiological concept is not one that readily fits into a system of quantities such as the ISQ. Every human eye is different, so the SI abandons the physiological definition and defines the unit, candela, as a fixed fraction of the radiant intensity received from a monochromatic light source of a fixed frequency and power. That leaves unanswered the question of the luminous intensity of light received from a source of a different frequency; it has to be obtained by applying an empirically determined 'standard luminosity function' (quoted in neither ISO 80000 nor the SI) that averages widely differing results obtained from real human beings by various means. The SI unit is of a quantity that differs from all other base quantities in that it includes arbitrarily in its definition an impersonal, empirically derived function that purports to represent a subjective quantity.

The standard luminosity function is dimensionless, so luminous intensity at a particular wavelength is defined as a fixed fraction of radiant intensity at that wavelength, a derived quantity in the ISQ. Before its adoption as a base unit the candela could have been called a non-coherent unit in the SI; indeed it is also that. Despite their undoubted usefulness and widespread employment by lighting engineers the lumen and all the units derived from the concept, including the candela, are not justifiably SI units. Acoustic engineers also have a set of units that almost parallel those of lighting engineers, including units associated with perceived loudness; but loudness, another physiological quantity, is not a base quantity of the ISQ.

A parallel might be drawn between the concepts of luminous intensity and weight. The weight of a particular mass, say a standard kilogram, is the force required to counter the force of gravity on it. That force varies according to where on the planate the mass is, because the acceleration due to gravity varies from place to place. The 'weight' can be made a property of the mass by redefining the term 'weight', by assuming a conventional, fixed, standard value of the 'acceleration due to gravity' unrelated to place, so that the mass has the same 'weight' everywhere. It would be its 'standard weight'. In the same way luminous intensity, a property that is personal to the owner of a human eye and that differs from person to person, is made instead a property of the light source, independent of that eye by replacing it by an invariable mathematical function, unrelated to the human eye that perceives the light. It might at least be renamed '*standard* luminous intensity'. I find it hard to imagine 'standard weight' ever being adopted as a base quantity of the ISQ, with its own SI unit.

Conclusion

Of the seven base quantities selected for the ISQ: length, time, electric current, mass, thermodynamic temperature, luminous intensity and amount of substance, only the first two, length and time, are unequivocally base quantities according to the VIM's definition. They are so familiar that they defy definition. The definition of mass relies on Newton's law and is thus not independent of length and time. It would be better replaced by force as a base quantity. Thermodynamic temperature is not a quantity according to VIM3's definition of the term; and it has no unit. It could be replaced by *interval of temperature*. Luminous intensity is an empirically derived function that purports to represent a subjective quantity. Amount of substance is not a quantity in accordance with the term's definition of quantities with units.

References

SI8 2006 *The International System of Units (SI)* 8th edn (Paris: Bureau International des Poids et Mesures);

VIM3 2008 *International Vocabulary of Metrology—Basic and General Concepts and Associated Terms, VIM* 3rd edn JCGM 200:2008 (Paris: Bureau International des Poids et Mesures);

ISQ ISO/IEC 80000 International Organization for Standards, Geneva;

IUPAC *The Gold Book*, International Union of Pure and Applied Chemistry, North Carolina, USA

IUPAP International Union of Pure and Applied Physics

New SI BIPM Sèvres

ITS-90 httpp//www.its-90.com/its-90.html;

[i] Coins are minted so that each coin of a particular denomination is countable. It is made to exacting specifications of linear dimensions and mass. Banks have machines that can measure those quantities and use the results to sort a mixed bag of coins into their respective denominations: and count them. The result for each denomination is a number and a face value. But the coins themselves are not quantities. In the same way molecules of a particular substance may have many different properties, but they themselves are not properties. They are not quantities. They are not units.

[ii] In counting entities it is sufficient to recognise each entity as being of the defined class. Its mass, for example, is irrelevant, even if the definition of the class requires the members to have a masses that fall into a given range..